

Further Pure 1 - June 2007

$$\begin{aligned} \textcircled{1} \text{ a) } M &= \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} \\ &= 3 \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \end{aligned}$$

b) Enlargement SF 3, (so $p = 3$), reflection in line $y = -x$

$$\begin{aligned} \text{c) } M^2 &= \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} &= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} = 9I \end{aligned}$$

$$\textcircled{2} \text{ a) Let } f(x) = x^3 + x - 7$$

$$f(1.6) = 1.6^3 + 1.6 - 7 = -1.304$$

$$f(1.8) = 1.8^3 + 1.8 - 7 = 0.632$$

Sign change, \therefore root between 1.6 and 1.8

$$\text{b) } f(1.7) = 1.7^3 + 1.7 - 7 = -0.387$$

\therefore root lies between 1.7 and 1.8

$$f(1.75) = 1.75^3 + 1.75 - 7 = 0.1693\dots$$

\therefore root lies between 1.7 and 1.75

\therefore root = 1.7 (1dp)

$$\textcircled{3} \text{ a) } z = 3iz^*$$

$$= (x + iy) = 3i(x - iy)$$

$$= x + iy = 3xi - 3y \quad (i^2 = -1)$$

$$= x - 3y = 3xi + iy$$

REAL	$x - 3y$
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IMAG	$-3x + y$
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b) REAL $x - 3y = 16$ (1)

IMAG $-3x + y = 0$ (2)

(1) $\times 3 \rightarrow 3x - 9y = 48$

(2) $-3x + y = 0$

$-8y = 48$

$\rightarrow y = -6$

use (1): $x - 3(-6) = 16 \rightarrow x = -2$

$\therefore z = -2 - 6i$

(4) a) $\alpha + \beta = -(-1/2) = 1/2$ $\alpha\beta = 4/2 = 2$

b) $1/\alpha + 1/\beta = \beta/\alpha\beta + \alpha/\alpha\beta$
 $= \frac{\alpha + \beta}{\alpha\beta} = \frac{1/2}{2} = 1/4$

c) Sum $4/\alpha + 4/\beta = 4(1/\alpha + 1/\beta) = 4(1/4) = 1$

Product $4/\alpha \times 4/\beta = 16/\alpha\beta = 16/2 = 8$

$\rightarrow x^2 - \text{Sum}x + \text{Product} = 0$

$\rightarrow x^2 - x + 8 = 0$

(5) a)

x	1	2	3	4
y	0.584	0.788	0.992	1.196

b) $y = ab^x$

$\log_{10}(y) = \log_{10}(ab^x)$

$\rightarrow \log_{10}(y) = x \log_{10}(b) + \log_{10}(a)$

$\rightarrow y = mx + c$
 $\downarrow \log_{10}(b)$ $\downarrow \log_{10}(a)$

c) See sheet

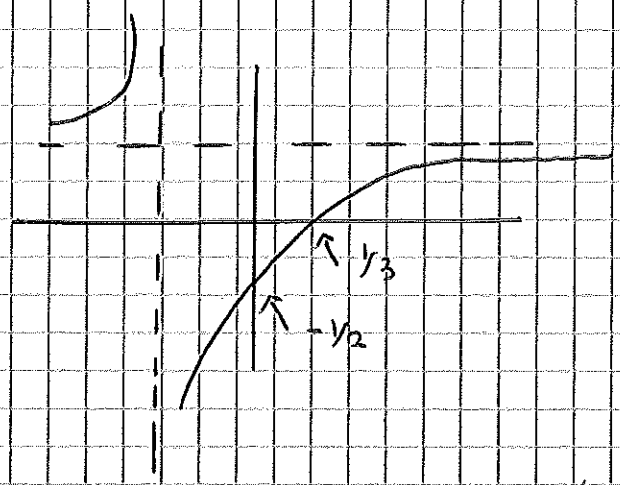
d) From graph's gradient ≈ 0.2 $\left. \begin{array}{l} \text{y intercept } \approx 0.38 \\ \therefore \log_{10}(b) \approx 0.2 \\ \rightarrow b \approx 10^{0.2} = 1.6 \end{array} \right\} \therefore \log_{10}(a) \approx 0.38$
 $\rightarrow a = 10^{0.38} = 2.4$

6) G.S $\theta = 2n\pi + \alpha$, $\theta = 2n\pi + (\pi - \alpha)$
 key angle (α) = $\sin^{-1}(\sqrt{3}/2) = \pi/3$
 $\rightarrow 2x - \pi/2 = 2n\pi + \pi/3$, $2x - \pi/2 = 2n\pi + 2\pi/3$
 $\rightarrow 2x = 2n\pi + 5\pi/6$, $2x = 2n\pi + 7\pi/6$
 $\rightarrow x = n\pi + 5\pi/12$, $x = n\pi + 7\pi/12$

7) a) Denominator = 0 when $x = -2 \rightarrow x = -2$
 As $x \rightarrow \infty$, $y \rightarrow 3/1 \rightarrow y = 3$

b) crosses x axis when $y = 0$
 $\rightarrow 0 = \frac{3x-1}{x+2} \rightarrow 0 = 3x-1 \rightarrow x = 1/3$

crosses y axis when $x = 0$
 $\rightarrow y = \frac{3(0)-1}{0+2} \rightarrow y = -1/2$



c) From diagram, graph is between 0 and 3 when $x > 1/3$

8) a) $\int_0^1 x^{1/3} + x^{-1/3}$ $\rightarrow \int_p^1 x^{1/3} + x^{-1/3}$
 $= \left[\frac{3}{4} x^{4/3} + \frac{3}{2} x^{2/3} \right]_p^1$
 $= 3/4 + 3/2 - \left[\frac{3}{4} p^{4/3} + \frac{3}{2} p^{2/3} \right]$
 $= 9/4 - \left[\frac{3}{4} \sqrt[3]{p^4} + \frac{3}{2} \sqrt[3]{p^2} \right]$
 As $p \rightarrow 0$, $\sqrt[3]{p^4}$ and $\sqrt[3]{p^2} \rightarrow 0$
 $\rightarrow \int \rightarrow 9/4$

$$b) \int_0^1 \frac{x^{1/3} + x^{-1/3}}{x} dx = \int_p^1 x^{-2/3} + x^{-4/3} dx$$

$$= \left[3x^{1/3} - 3x^{-1/3} \right]_p^1$$

$$= 3(1) - 3(1) - \left[3\sqrt[3]{p} - \frac{3}{\sqrt[3]{p}} \right]$$

As $p \rightarrow 0$, $\frac{3}{\sqrt[3]{p}}$ is undefined
 $\therefore \int$ has no value.

9) a) Intersects x axis when $y = 0$

$$\rightarrow \frac{x^2}{2} = 1 \rightarrow x^2 = 2 \rightarrow x = \pm \sqrt{2}$$

$(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$

Intersects y axis when $x = 0$

$$\rightarrow y^2 = 1 \rightarrow y = \pm 1 \rightarrow (0, 1) \text{ and } (0, -1)$$

b) $\frac{(x-k)^2}{2} + y^2 = 1$

c) $x + y = 2 \rightarrow y = 2 - x$

sub into translated curve

$$\rightarrow \frac{(x-k)^2}{2} + (2-x)^2 = 1$$

$$\rightarrow (x-k)^2 + 2(2-x)^2 = 2$$

$$\rightarrow x^2 - 2kx + k^2 + 2[4 - 4x + x^2] = 2$$

$$\rightarrow x^2 - 2kx + k^2 + 8 - 8x + 2x^2 = 2$$

$$\rightarrow 3x^2 - 2kx - 8x + k^2 + 6 = 0$$

$$\rightarrow 3x^2 - 2(k+4)x + (k^2+6) = 0$$

d) For tangent, $b^2 - 4ac = 0$

$$\rightarrow [-2(k+4)]^2 - 4(3)(k^2+6) = 0$$

$$\rightarrow (-2k-8)^2 - 12k^2 - 72 = 0$$

$$\rightarrow 4k^2 + 32k + 64 - 12k^2 - 72 = 0$$

$$\rightarrow -8k^2 + 32k - 8 = 0$$

$$\rightarrow k^2 - 4k + 1 = 0$$

$$\rightarrow (k - 2)^2 - 4 + 1 = 0$$

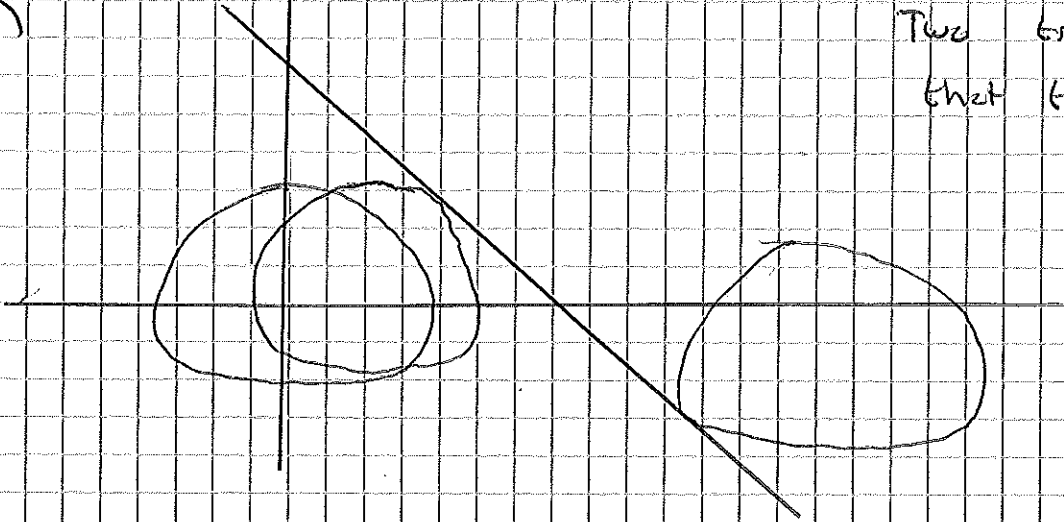
$$\rightarrow (k - 2)^2 - 3 = 0$$

$$(k - 2)^2 = 3$$

$$k - 2 = \pm\sqrt{3}$$

$$\rightarrow k = 2 \pm\sqrt{3}$$

e)



Two translated curves
that touch line once!